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DYNAMIC FLUID FLOW BEHAVIOUR OF A TANK DRAINING THROUGH A VERTICAL TUBE

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Abstract—This paper describes the unsteady draining of a sealed tank partially filled with water. The water discharges via a vertical tube into an open tank at atmospheric conditions. The air inflow, compensating for the volume of the discharged liquid, enters the system in an oscillatory manner, much like the "gulping" seen in an upended beer bottle. A mathematical model, based closly on that derived by Dougall & Kathiresan [*Chem. Engng Commun.* **8**, 289–304 (1981)], has been applied to predict the pressure fluctuations in the closed tank. The rate of water discharge from the tank has been predicted and gives a much closer agreement with experimental results than a prediction based on a steady counter-current flooding limitation approach. A drift flux model has been used to describe the two-phase flow effect in the tube and the Wallis flooding criterion has been modified for use in the slug flow regime to describe the boundary conditions at the bottom of the tube. The pressure fluctuations in the sealed tank have been measured and compared with results obtained from the mathematical prediction for a variety of tube diameters.

Key Words: two-phase flow, pressure fluctuation, flooding, oscillatory flow, draining tank, bubbles, gulping

1. INTRODUCTION

One aspect of the discharge of a fluid from a vessel via a vertical pipe is recognized by all who have poured themselves a drink from an upended bottle, where the inflow of air, which makes up for the volume of the discharged liquid, restricts the discharge by an oscillatory process which may be called "gulping". Here the rate of discharge is markedly reduced during the time air ingestion occurs. A literature review shows that the mathematical or physical models for computing discharges from tanks when air is ingested through the outlet have received little attention, although the somewhat related problem of the condition required for a liquid discharge pipe to run full has been treated by Wallis *et al.* (1977) and draining of punctured tanks without vacuum-relief has been considered by Dodge & Bowles (1982). In practice, excepting the work of Dougall & Kathiresan (1981), the available models take no account of the gulping nature of the discharge.

The intention of the present work is to determine the rate of fluid discharge and the pressure fluctuations associated with the transient behaviour of a tank draining via a vertical tube. This problem is of importance in many aspects of two-phase flow and especially in the safety analysis of the counter-current flooding limitations in pressurized water nuclear reactors (PWR).

This study is part of a wider investigation into oscillatory behaviour of transient counter-current two-phase flow in horizontal and vertical tubes which is relevant to loss of coolant accidents in PWR systems (Tehrani 1992).

2. EXPERIMENTAL APPARATUS

The experimental setup consisted of water flow from a partially-filled sealed tank measuring $25 \times 20.2 \times 15.2$ cm through a single vertical tube into an open tank at atmospheric pressure. The pressure in the sealed tank was measured by means of a differential pressure transducer. The signal from the transducer was amplified and converted into digital form using an A–D converter. A series interface was used to send the data to a PC for processing. The data were analysed using a fast Fourier transform algorithm to obtain the frequency response of the system. Experiments were carried out with various tube internal diameters between 9.0 and 25.4 mm with length 600 mm. The

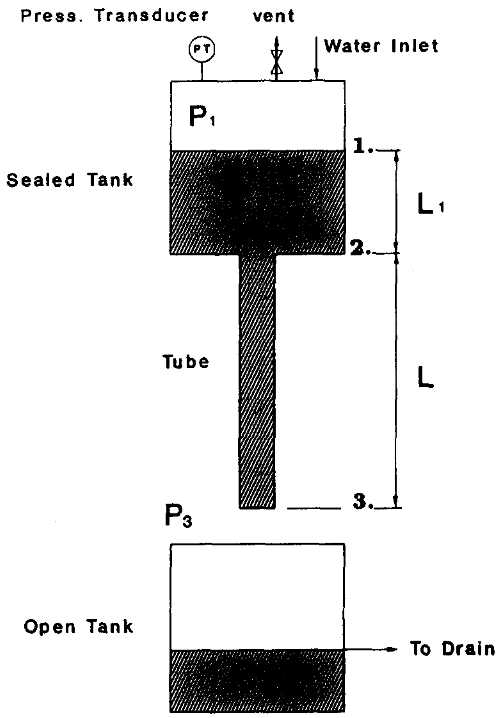

Figure 1. The experimental setup.

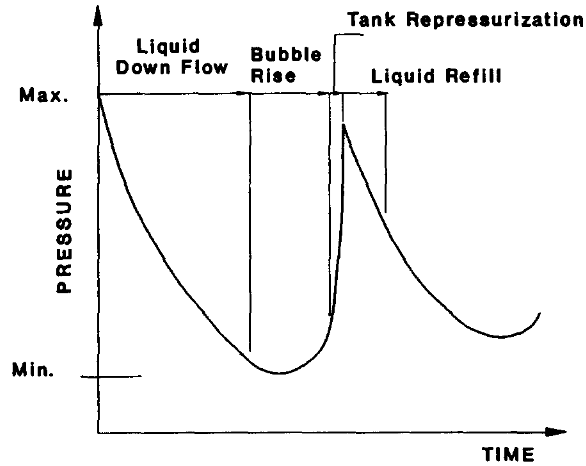

Figure 2. Pressure variation in the tank.

height of the liquid in the sealed tank was continuously recorded. The experimental arrangement is shown in figure 1.

3. CYCLIC DISCHARGE BEHAVIOUR

To describe the unsteady draining process of a sealed tank four distinct stages, as shown in figure 2, are considered. The four stages are:

> *Liquid downflow.* The liquid from the sealed tank flows down the tube thus causing a reduction in the air pressure above the liquid surface. When the pressure falls sufficiently below atmospheric, the liquid velocity in the tube is reduced to almost zero and the next stage of the cycle begins.
> *Bubble rise.* An air bubble is formed at the exit of the tube and rises, filling the tube. When the bubble reaches the top of the tube the next stage commences.
> *Tank repressurization.* Air is ingested rapidly through the tube increasing the pressure in the tank to the point where liquid can enter the tube, thus leading to liquid refill.
> *Liquid refill.* Liquid refills the tube from the top until no air remains in the tube, reducing the air pressure in the closed tank, and the whole cycle is then repeated with slowly decreasing frequency.

It should be noted that figure 2 shows the first cycle of the process. Since at the beginning of the process the tube is full of water, the first liquid refill of the tube takes place at the start of the second cycle.

4. THEORETICAL DEVELOPMENT

The present mathematical model is closely based on the theoretical approach by Dougall & Kathiresan (1981), which was applied to the different but related geometry of flow though a vertical tube from a fixed head of water in an open upper tank to a sealed lower one. Consistency of terminology and nomenclature is observed wherever possible. A number of individual descriptions have been brought together to define each stage of the cycle. The model involves parameters averaged over the complete tube or the portion of the tube above or below the two-phase interface and the flow is assumed to be one-dimensional. The liquid is incompressible and the flow in the tube is fully developed. Each stage of the process is discussed below.

4.1. Liquid downflow model

Conservation of momentum for the liquid in the tube integrated between stations 2 and 3 (figure 1) is given by

$$\frac{\mathrm{d}V_\mathrm{L}}{\mathrm{d}t}L = -\left[\frac{1}{\rho_\mathrm{L}}(P_2 - P_3) + gL - \frac{\tau P_\mathrm{w} L}{\rho_\mathrm{L} A}\right], \qquad [1]$$

where V_L is the liquid velocity in the tube, τ is the tube wall shear stress, L and A are the length and the cross-sectional area of the tube, respecitvely, and P_w is the tube perimeter. The direction of the Z-coordinate is vertically upwards and velocities are considered positive in this direction. Further, using the one-dimensional energy equation between stations 2 and 1 (figure 1):

$$(P_2 - P_1) = \rho_\mathrm{L}\left[gL_1 - \frac{V_\mathrm{L}^2}{2}(1 + K_\mathrm{c})\right], \qquad [2]$$

where ρ_L is the liquid density and K_c is the sudden entry loss coefficient. From [1] and [2] the governing equation for the velocity of the liquid is

$$\frac{\mathrm{d}V_\mathrm{L}}{\mathrm{d}t} = -\left[\frac{(P_1 - P_3)}{\rho_\mathrm{L} L} + g\left(1 + \frac{L_1}{L}\right) - \frac{\tau P_\mathrm{w}}{\rho_\mathrm{L} A} - (1 + K_\mathrm{c})\frac{V_\mathrm{L}^2}{2L}\right], \qquad [3]$$

where

$$\tau = 0.5 C_\mathrm{f} \rho_\mathrm{L} V_\mathrm{L}^2,$$

C_f is the friction factor, and

$$\mathrm{Re} = \frac{\rho_\mathrm{L} D V_\mathrm{L}}{\mu_\mathrm{L}}$$

for

$$\mathrm{Re} < 2500 \qquad C_\mathrm{f} = \frac{16}{\mathrm{Re}}$$

and

$$\mathrm{Re} > 2500, \quad C_\mathrm{f} = \frac{0.079}{(\mathrm{Re})^{0.25}}.$$

The gas in the sealed tank is expanded as the liquid is drained from the tank. Assuming the expansion of the air in the sealed tank follows a polytropic process for a perfect gas and assuming $\rho_\mathrm{G} \ll \rho_\mathrm{L}$,

$$P_1 v_\mathrm{G}^n = C_l. \qquad [4]$$

Where P_1 is the pressure inside the tank, v_G is the volume of gas in the tank and C_l is the initial tank constant. The total volume in the tank is given by

$$v_\mathrm{t} = v_\mathrm{G} + v_\mathrm{L}. \qquad [5]$$

The rate of change of liquid volume in the sealed tank is determined by using [4] and [5]:

$$\frac{\mathrm{d}v_\mathrm{L}}{\mathrm{d}t} = \frac{(C_l)^{1/n}}{n(P_1)^{(n+1)/n}}\frac{\mathrm{d}P_1}{\mathrm{d}t}. \qquad [6]$$

The rate of change of liquid volume in the sealed tank is determined by the liquid velocity and the tube cross-sectional area:

$$\frac{\mathrm{d}v_\mathrm{L}}{\mathrm{d}t} = -V_\mathrm{L} A. \qquad [7]$$

Combining [6] and [7] gives

$$\frac{dP_1}{dt} = \frac{n(P_1)^{(n+1)/n}}{(C_l)^{1/n}} V_L A, \qquad [8]$$

where C_l is calculated from [4]:

$$C_l = P_1(\epsilon_t v_t)^n, \qquad [9]$$

where ϵ_t is the void fraction of the tank. The rate of change of the void fraction in the sealed tank depends on liquid velocity, tube diameter and the tank volume. Hence

$$\frac{d\epsilon_t}{dt} = V_L\left(\frac{A}{v_t}\right). \qquad [10]$$

Equations [3], [8] and [10] give a set of three first-order non-linear differential equations for the liquid velocity in the tube, pressure in the tank, P_1, and the void fraction in the tank. The liquid downflow will stop when the velocity of the liquid reaches the "bubble formation velocity" ($V_{B,form}$), reported by White & Beardmore (1962) as

$$V_{B,form} = \left(\frac{0.345}{C_0}\right)(gD)^{0.5}. \qquad [11]$$

The initial conditions for the first cycle were set at atmospheric pressure, an arbitrary value for velocity slightly higher than the bubble formation velocity and the initial void fraction in the tank. For subsequent cycles, the initial conditions for the pressure, liquid velocity and the tank void fraction were obtained from the previous liquid refill stage of the process.

4.2. Bubble rise model

In this stage the air forms a bubble in the tube and begins to rise, essentially filling the tube, figure 3. The behaviour lies in the slug flow regime and will be analysed using the drift flux model developed by Zuber & Findlay (1965). The drift flux relationships for the two-phase flow are:

$$j_G = \epsilon(C_0 j + V_{Gj}),$$
$$j_L = (1 - \epsilon C_0)j - \epsilon V_{Gj},$$
$$j = j_G + j_L; \qquad [12]$$

where j is defined as the volumetric flow rate divided by the cross-sectional area of the tube and ϵ is the void fraction in the tube. The equation for the bubble rise velocity is

$$\frac{dZ_b}{dt} = V_b = \frac{j_g}{\epsilon} = (C_0 j + V_{Gj}). \qquad [13]$$

The mass of gas in the tube in terms of the tube void fraction and cross-sectional area is

$$M_G = \rho_G \int_0^{Z_b} \epsilon A \, dZ. \qquad [14]$$

The rate of change of mass of gas in the tube is

$$\frac{dM_G}{dt} = \rho_G j_G A. \qquad [15]$$

From [14] and [15],

$$\frac{d}{dt}\left(\int_0^{Z_b} \epsilon \, dZ\right) = (j_G)_0. \qquad [16]$$

The combined momentum equation for both phases of a slip flow in a vertical channel is given by

$$\frac{\partial}{\partial t}(\rho_G j_G + \rho_L j_L) + \frac{\partial}{\partial Z}\left(\rho_G \frac{j_G^2}{\epsilon} + \rho_L \frac{j_L^2}{(1-\epsilon)}\right) = -\frac{\partial P}{\partial Z} - g[\rho_G \epsilon + (1-\epsilon)\rho_L]. \qquad [17]$$

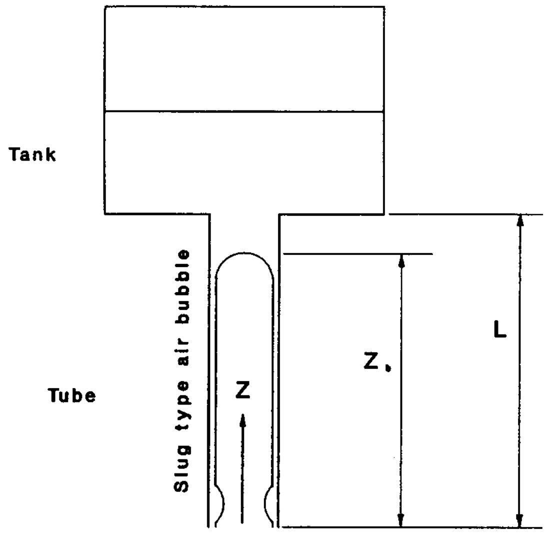

Figure 3. Slug flow through the tube.

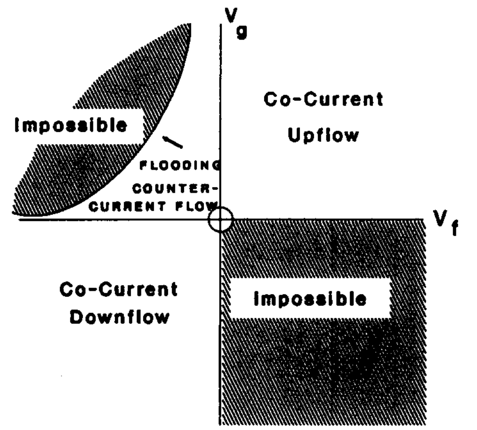

Figure 4. Region of possible operation based on a simplified one-dimensional flow treatment (Hewitt 1978).

Wall friction is neglected in this equation because the relatively slow bubble rise leads to small liquid velocities close to the wall.

For $0 < Z < Z_b$,

$$j_G = \epsilon(C_0 j + V_{Gj}),$$
$$j_L = (1 - \epsilon C_0 j) - \epsilon V_{Gj},$$
$$j = j_G + j_L;$$

and for $Z_b < Z < L$,

$$j_G = 0,$$
$$j_L = j. \tag{18}$$

Let

$$j = j(t),$$
$$\epsilon = \epsilon(Z, t). \tag{19}$$

Integrating [17] over the length of the tube L, using [18] and [19] and simplifying for the volumetric flux, gives

$$\left[\rho_L L - C_0(\rho_L - \rho_G)\int_0^{Z_b} \epsilon \, dZ\right]\frac{dj}{dt} = (P_3 - P_2) - \rho_L g\left[L + \left(\frac{\rho_G}{\rho_L} - 1\right)\int_0^{Z_b} \epsilon \, dZ\right] + (\rho_L - \rho_G)(C_0 j + V_{Gj})(j_G)_0 - \rho_L j^2 + \rho_G \frac{(j_G)_0^2}{\epsilon_0} + \rho_L \frac{(j - j_G)_0^2}{(1 - \epsilon_0)}. \tag{20}$$

The same result may be obtained from a simple force-momentum balance over the tube and is slightly different from that reported by Dougall & Kathiresan (1981). It is thought that there is a minor error in their formulation. It should also be noted that the third term on the RHS of their corresponding equation becomes equal to zero for the present case as air enters the tube from the atmosphere.

From [4], the rate of change of pressure in the tank is given by

$$\frac{\mathrm{d}P_t}{\mathrm{d}t}=\frac{njAP_t}{\epsilon_t v_t}. \qquad [21]$$

The equation for the void fraction in the sealed tank is

$$\frac{\mathrm{d}\epsilon_t}{\mathrm{d}t}=j_L\frac{A}{v_t}$$

or

$$\frac{\mathrm{d}\epsilon_t}{\mathrm{d}t}=(j-j_G)\frac{A}{v_t}. \qquad [22]$$

The flooding condition, the counter-current flow limitation, is considered to provide a method of evaluating the boundary conditions at the bottom of the tube. The flooding relationship developed by Wallis (1969) is used:

$$(j_G^*)^{1/2}+m(-j_L^*)^{1/2}=C, \qquad [23]$$

where

$$j_G^*=j_G\left[\frac{\rho_G}{(\rho_L-\rho_G)gD}\right]^{1/2} \quad \text{and} \quad j_L^*=j_L\left[\frac{\rho_L}{(\rho_L-\rho_G)gD}\right]^{1/2}.$$

The flooding equation can be rewritten in terms of j_L and j_G:

$$(j_G)^{1/2}+m\left(\frac{\rho_L}{\rho_G}\right)^{1/4}(j_L)^{1/2}=C(gD)^{1/4}\left(\frac{\rho_L}{\rho_G}-1\right)^{1/4}. \qquad [24]$$

The operational counter-current flooding curve is shown in figure 4 and the slope of this curve is given by

$$\frac{\mathrm{d}j_G}{\mathrm{d}j_L}=\frac{-\left[m\left(\frac{\rho_L}{\rho_G}\right)^{0.25}\right]^2(j_G)^{0.5}}{j_G^{0.5}-C(gD)^{0.25}\left(\frac{\rho_L}{\rho_G}-1\right)^{0.25}}. \qquad [25]$$

The drift flux relationships of [12] can be rearranged in terms of j_L and j_G:

$$j_G=\frac{\epsilon C_0}{(1-\epsilon C_0)}j_L+\frac{\epsilon}{(1-\epsilon C_0)}V_{Gj}. \qquad [26]$$

Differentiating [26] and substituting for $\mathrm{d}j_G/\mathrm{d}j_L$ gives an equation for the drift flux velocity:

$$V_{Gj}=C_0\left(-j_L+\frac{j_G}{\left(\frac{\mathrm{d}j_G}{\mathrm{d}j_L}\right)}\right). \qquad [27]$$

Hence the void fraction in the tube below the nose of the bubble can be calculated from [12] and [27]:

$$\epsilon=\frac{j_G}{(C_0 j+V_{Gj})}$$

or

$$\epsilon=\frac{1}{C_0\left[1+\frac{1}{\left(\frac{\mathrm{d}j_G}{\mathrm{d}j_L}\right)}\right]}. \qquad [28]$$

Equation [24] is solved numerically using Newton's iterative method to evaluate j_G, and to calculate [25] and [28]. It is then possible to solve the five differential equations [13], [16] and [20]–[22].

The initial conditions for this stage of the process are as follows:

$$j_G = 0.0,$$

$$Z_b = 0.0,$$

$$\int_0^{Z_b} \epsilon \, dZ = 0.0.$$

The pressure and tank void fraction are those of the end of the liquid refill stage. The bubble rise stage of the cycle is completed when Z_b reaches the total length of the tube L.

4.3. Tank repressurization model

In this stage of the process the air in the vertical tube is released into the tank. Since this stage is so rapid, it is assumed that no liquid exists in the tube during the repressurization phase.

The rate of change of pressure in the tank is given by

$$\frac{dP_1}{dt} = \frac{nP_1 V_G A}{\epsilon_t v_t}. \qquad [29]$$

The initial conditions are taken from the bubble rise stage of the process. From a simple energy balance the gas velocity in the tube is given by

$$V_G = \left[\frac{2(P_3 - P_2)}{(1 + K_c)\rho_G}\right]^{1/2}, \qquad [30]$$

where

$$P_2 = P_1 + \rho_L g L_1.$$

In [30], it is assumed that the gas flowing in the tube discharges freely into the tank, where there is an entry loss coefficient K_c, and no recovery of energy of the gas leaving the tube.

Wallis (1969) related the critical gas velocity at which no liquid could run down a tube by the empirical relationship

$$V_{G\,max} = 0.525(\rho_G)^{-1/2}[gD(\rho_L - \rho_G)]^{1/2}. \qquad [31]$$

Hence, from [30]:

$$P_1 = (P_3 - \rho_L g L_1) - 0.1375 gD(1 + K_c)(\rho_L - \rho_G). \qquad [32]$$

Equation [32] controls the boundary condition at the end of the process.

4.4. Liquid refill model

In this part of the process, the pressure in the tank has now increased to such a value that the liquid can enter the top of the tube, refilling it until no air remains. The rate of change of pressure in the tank is given by

$$\frac{dP_1}{dt} = \frac{nP_1 V_L A}{\epsilon_t v_t}. \qquad [33]$$

The rate of change of bubble length in the tube is related to velocity by

$$\frac{dZ_G}{dt} = V_L. \qquad [34]$$

The rate of change of velocity in the tube can be derived using the force–momentum equation:

$$[\rho_L L - (\rho_L - \rho_G)Z_G]\frac{dV_L}{dt} = (P_3 - P_2) - g[\rho_L L - (\rho_L - \rho_G)Z_G] + \frac{\rho_G V_L^2 Z_G A}{\epsilon_t v_t}. \qquad [35]$$

The final term of [35] accounts for the variation of gas density as a function of time and the friction term has been neglected. The pressure, P_2, at station 2 is given by

$$P_2 = P_1 + \rho_L g L_1. \quad [36]$$

In [36], L_1 is the height of liquid in the sealed tank and is calculated by

$$L_1 = \frac{(1-\epsilon_t)v_t}{A_{t,\,base}}, \quad [37]$$

where $A_{t,\,base}$ is the base area of the tank.

The liquid velocity (V_L) is set to zero for the initial condition of this stage. This stage of the process ends when liquid fills the length of the tube ($Z_G = 0.0$) and liquid downflow begins. The cycle then repeats itself.

5. RESULTS AND DISCUSSION

All the differential equations were integrated numerically using the fourth-order Runge–Kutta–Merson method. The flooding criterion of [23] for a sharp-edged tube was applied specifically for the slug flow regime, yielding values of $m = 0.6$ and $C = 0.63$, which differ from the values averaged over a range of flow regimes used by Dougall & Kathiresan (1981). Considering the time scale involved in the expansion of the air in the tank, the process was assumed to be isothermal, therefore the polytropic index of 1 was chosen for this work. To be able to make comparisons between present work and that of Dougall & Kathiresan (1981) it was decided to maintain the distribution parameter, C_0, as 1 instead of the more usually accepted value of 1.2. However, use of the latter would marginally improve the accuracy of the results obtained by the present work. The parameters used in the experimental and theoretical modelling are listed in table 1.

The pressure fluctuations in the sealed tank were recorded experimentally and these are compared with the calculated pressure variations for 9.0, 15.9 and 19 mm tube diameters in figures 5, 6 and 7, respectively, together with their frequency spectra. The general shape of the calculated values [parts (a)] is in good agreement with the experimentally measured results [parts (b)]. However, the theoretical model predicts a cycle period which is approx. 5% shorter than the experimentally measured values. The small instabilities which occur during the experimental runs are not predicted in the theoretical approach and this increases the total inaccuracy to about 10%. This indicates that the present calculation is an improvement on that of Dougall & Kathiresan (1981) who found errors of some 15% in the prediction of the cycle period. These improvements are chiefly due to the more appropriate calculation of wall shear stress in the laminar and turbulent regimes in the liquid downflow region, use of the relevant flooding constants for the slug flow process and, more significantly, corrected and modified equations in the bubble rise stage of the process.

To identify the characteristic frequency of each system the frequency spectrum of the pressure fluctuations in the tank was obtained and typical results are shown in figures 5–7 [parts (c)]. It may be noted that the characteristic frequency increases approximately linearly with increase in the tube diameter but, no firm conclusion can be drawn because of the limited range of data available in this work. The presence of harmonics in the frequency plots is an indication of instabilities in the process and is exaggerated because of the small number of samples involved in each run.

Figure 8 shows the predicted cyclic growth and collapse of the gas bubbles. This plot indicates that the bubble rise stage occupies a comparatively short portion of the total process—somewhat

Table 1. Parameters used in the calculations

$L = 0.6\ \mathrm{m}$	$D = 9.0$ to $25.0\ \mathrm{mm}$
$\rho_L = 1000\ \mathrm{kg\,m^{-3}}$	$\rho_G = 1.2\ \mathrm{kg\,m^{-3}}$
$v_t = 7.688 \times 10^{-3}\ \mathrm{m^3}$	$K_c = 0.5$
$m = 0.6$	$n = 1.0$
$C = 0.63$	$C_0 = 1.0$

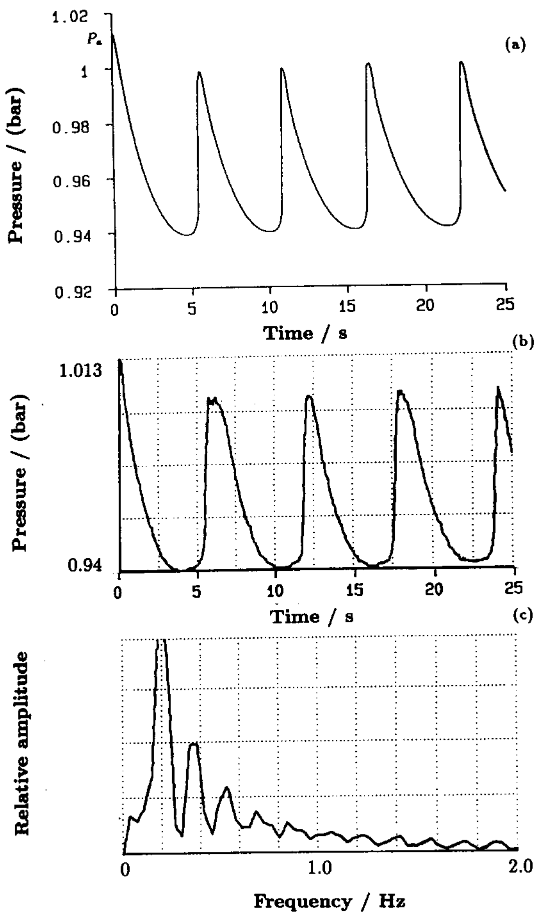

Figure 5. Comparison of calculated (a) and measured (b) pressure in the closed tank draining through the 9.0 mm tube with spectral analysis of the signals from the pressure transducer (c).

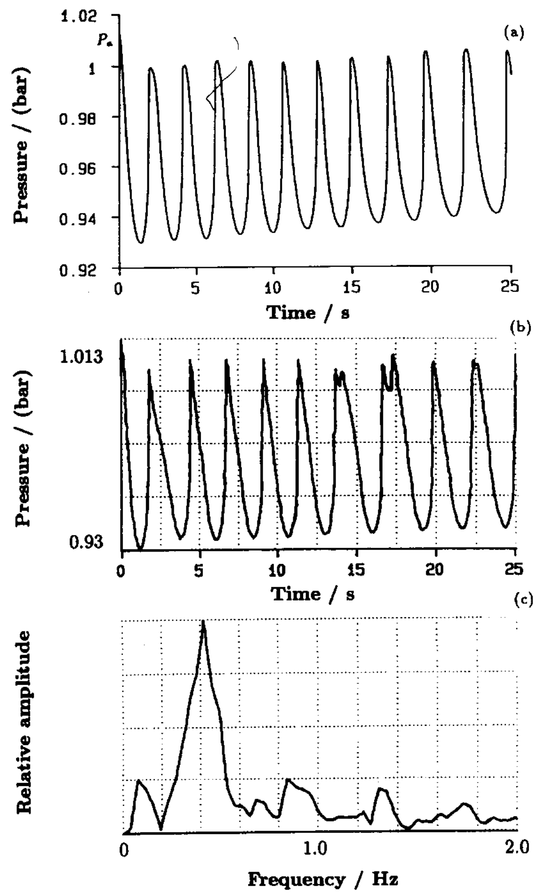

Figure 6. Comparison of calculated (a) and measured (b) pressure in the closed tank draining through the 15.9 mm tube with spectral analysis of the signals from the pressure transducer (c).

less than 20% of the cycle time. It is only during the bubble rise stage that the steady counter-current flooding limitation (CCFL) controls the flow rate and during this time very little liquid drains from the tank. The theoretical and experimental variation of the void fraction in the closed tank draining through the 15.9 mm tube is shown in figure 9. The plot indicates a good correlation between the predicted and the experimental values; however, discrepancies arise due to random instabilities within cycles which are not predicted in the model.

It was observed experimentally that the time taken for each cycle increased as the tank emptied. This is due to the fact that, with each cycle, a certain amount of water flows out of the tank, the air enters and the volume available for expansion in the next cycle is increased. Hence the air is expanded less rapidly in the next cycle. This allows more water to discharge and increases the period of the cycle. However, the cycle time increase is small and non-linear with respect to the air volume increase. This behaviour is predicted by the present theoretical model.

It was also observed that, for most of the tube diameters tested, it takes roughly the same number of cycles to drain a given amount of water from the tank independent of the discharge area. However, for the 25.4 mm dia tube, the bubbles did not fill the whole pipe and the pattern was no longer slug flow. The regular oscillations in the system were suppressed by a continuous, churning, counter-current flow in the tube and the flow rate out of the tube became erratic. In later experiments shorter lengths of the 25.4 mm dia tube were used to correspond to tube volume/tank volume ratios equivalent to those for the 19.6 and 15.9 mm tubes, respectively. For these it was found that a regular oscillatory pattern emerged but, in these cases, the liquid refill stage of the process was not able to reach the full length of the drain tube. It is intended to investigate further

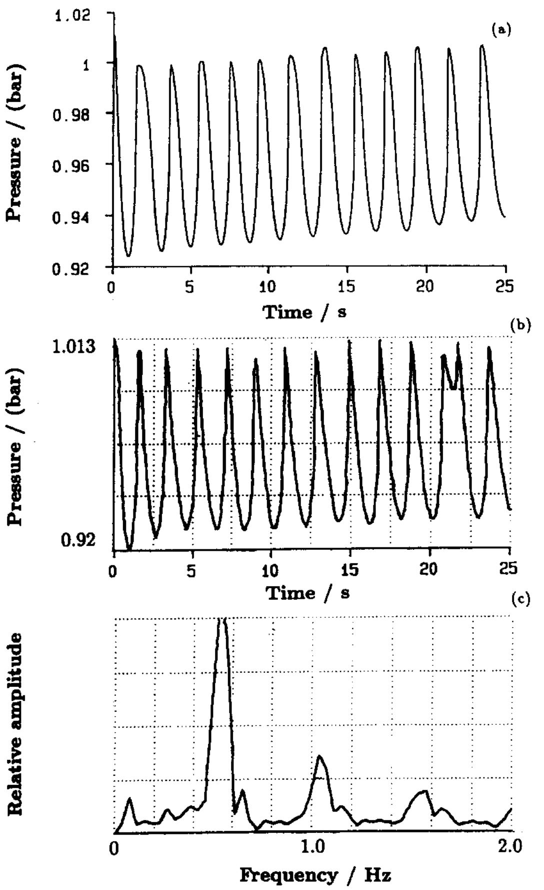

Figure 7. Comparison of calculated (a) and measured (b) pressure in the closed tank draining through the 19.6 mm tube with spectral analysis of the signals from the pressure transducer (c).

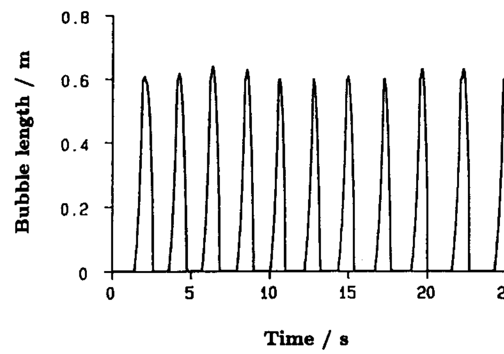

Figure 8. Predicted variation of gas bubble length with time for the 15.9 mm tube.

the possibilities of oscillatory counter-current flow for a wider range of tank and tube volumes. Dougall & Kathiresan (1981) reported that, for their 9.5 mm tube, surface tension dominated the process and the bubble did not rise at all. They reported an operating Eötvos number (ratio of buoyancy forces to the surface tension) of 6.0, which is close to the critical value of 4 given by White & Beardmore (1962) as the limit below which the bubble does not rise. Such difficulties were not experienced in the present work, since the Eötvos number for the 9.0 mm tube was of the order of 10. It is however acknowledged that factors such as tube surface roughness and cleanliness might have had some effects on the critical Eötvos number.

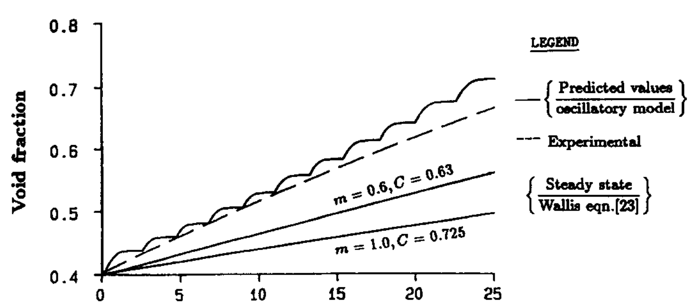

Figure 9. Comparison between exprimental and, both steady- and unsteady-state models, for theoretical void fraction variation in the sealed tank draining through the 15.9 mm tube.

To compare the variation with time of the void fraction in the sealed tank for the oscillatory flooding condition with that assuming steady counter-current flooding, [23] of Wallis (1961) has been used. Since the tank is emptied to atmospheric conditions, the process can be assumed to involve equal volumetric flows of air and water. For this situation the ratio of the superficial velocities of the two phases is thus proportional to the density ratio of the two fluids. This results in $(j_G^*)^{1/2}$ being 0.186 times $(j_L^*)^{1/2}$, so that the steady-state CCFL gives

$$j_L^* = \frac{C^2}{(m + 0.186)^2}, \qquad [38]$$

where the values of C and m depend on the detailed flow regime under which flooding takes place.

The values of j_L^* for two particular flooding conditions have been used to calculate the time variation of the tank void fraction on a non-gulping basis and the results are shown in figure 9. Two straight lines represent use of the flooding parameters proposed by Wallis (1969) ($m = 1.0$ and $C = 0.725$) and those used in the present calculations ($m = 0.6$ and $C = 0.63$) and are compared with results from the full model (including gulping) and the "smoothed" variation of the tank void fraction obtained from the present experiments. This clearly demonstrates that use of the steady-state CCFL model gives a large over-prediction of the tank emptying time compared with the oscillatory model, which takes account of the gulping phenomena and agrees reasonably closely with the experimental results.

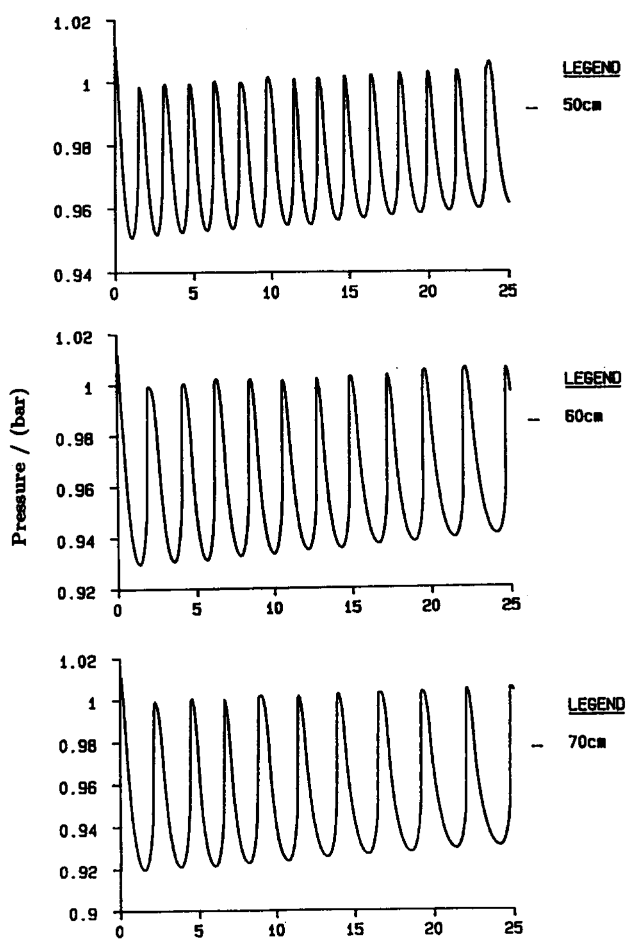

Figure 10. Comparison of predicted pressure variation in the sealed tank draining through the 15.9 mm dia tube for different tube lengths.

As a further demonstration of the use of the model, the theoretically predicted influence of tube length on cycle time is shown in figure 10. This clearly suggests that increase in the tube length results in an increase in the cycle period and further experiments are planned to test this prediction.

6. CONCLUSION

A mathematical model has been developed to predict the pressure fluctuations in a draining, closed tank. This agrees well with experimental data, which demonstrate the oscillatory counter-current flow which gives rise to the fluctuations. Such oscillations dominate the system behaviour and reduce the significance of the stable CCFL condition, as this occupies only a small portion of the cycle time.

In the experiments, for tube diameters above 25.0 mm, the flow was no longer oscillatory and a continuous churning counter-current flow was observed. This is not predicted by the present model which makes its application limited to tube sizes between 8.0 and 25.0 mm dia. However, it should be noted that the calculations apply for air and water at atmospheric pressure and the range of applicability is likely to be different for water and steam at higher pressures, as may be encountered in some industrial cooling systems.

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